Worksheet 1

1. Truth Tables

a. Fill in the following truth table

P	Q	$P \Longrightarrow Q$	$Q \Longrightarrow P$	$(P \Longrightarrow Q) \wedge (Q \Longrightarrow P)$
T	T			
T	F			
F	T			
F	F			

Fact: $(P \Longrightarrow Q) \wedge (Q \Longrightarrow P) \equiv P \Leftrightarrow Q$

b. Use the above truth table to determine the truth value for each of the following biconditional sentences

i. $1 + 1 = 2$ if and only if $cos(\pi) = -1$

ii. The moon is made of cheese if and only if the earth is flat

c. Use truth table to prove that $P \Longrightarrow Q \equiv (\neg Q) \Longrightarrow (\neg P)$

P	Q	$\neg Q$	$\neg P$	$P \Longrightarrow Q$	$(\neg Q) \Longrightarrow (\neg P)$
T	T				
T	F				
F	T				
F	F				

How is $(\neg Q) \Longrightarrow (\neg P)$ related to $P \Longrightarrow Q$? What can you say about it?

d. Construct a truth table for $(P \Longrightarrow Q) \wedge (Q \Longrightarrow R)$

2. Converse Vs. Inverse Vs. Contrapositive

   Write the converse, inverse, and contrapositive of each statement. And determine the truth value of the original statement, its converse, its inverse, and its contrapositive

   a. If f is an even function, then $f(2) = f(-2)$
      i. Converse

      ii. Inverse

      iii. Contrapositive

   b. If $x > 1$, then x is positive
      i. Converse

      ii. Inverse

      iii. Contrapositive

   c. If the real number $\sqrt{5}$ is rational, then π is rational
      i. Converse

      ii. Inverse

      iii. Contrapositive

3. Negating Quantified Statements

   Step 1: Decide whether the statement is true or false. If false, give a counterexample.

   Step 2: Convert English sentence (as much as possible) into a formulaic statement.

   Step 3: Negate the statement and write the negation in both forms

   a. The square of every real number is positive

      i. True / False:

      ii. Formulaic Statement:

      iii. Negation:

   b. Every p-series is convergent

      (Recall from MATH 143: A p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$)

      i. True / False:

      ii. Formulaic Statement:

      iii. Negation:

c. For every integer n, $4n + 1$ is odd

   i. True / False:

   ii. Formulaic Statement:

   iii. Negation:

d. There exists a natural number n such that $2^n - 1$ is prime

   i. True / False:

   ii. Formulaic Statement:

   iii. Negation:

e. For every positive real number ε, there exists a natural number N such that $\frac{1}{N} < \varepsilon$

   i. True / False:

   ii. Formulaic Statement:

   iii. Negation: